PERFORMANCE EVALUATION FOR DARCY FRICTION FACTOR FORMULAE USING COLEBROOK-WHITE AS REFERENCE

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ABSTRACT

Estimation of Darcy friction factor and pipe network analysis are essential ingredients in the design and distribution of potable water. Common formulae for friction factors estimate include Colebrook-White, Moody, Swamee and Jain, Barr, Haaland, Tsal and Wood formulae. Accuracy of pipe network analysis depends on Darcy friction factor, but little is known on update of these formulae and their performance in developing countries. In this paper, as a follow up on our previous studies, Oke (2007); Babatola *et al.* (2008) an overview and performance evaluation of these formulae is presented using statistical methods (model of selection criterion and statistical errors). Darcy Friction factor formulae were obtained from archive. These formulae were used to estimate friction factors in pipes at various Reynolds number and relative roughness. Estimated friction factors were evaluated statistically using absolute error, total error, mean error and model of selection criterion using Colebrook-White friction factor as the reference friction factor. Colebrook-White was used as reference because it is widely recommended formulae and has a wide range of Reynolds number. The study revealed that friction factor in pipes varies with the formulae and varies from 0.0157 to 0.0727. In all cases Tsal formula has the smallest friction factors. Based on the mean error, accuracy were in order of Newton Raphson > Prandtl and Nikurdse > Zingrang and Sylvester > Serghide > Barr > Swamee and Jain > Eck > Haaland > Brkic > Wood > Moody > Chen (1979) > Buzzelli > Sonnad and Goudar > Vatankhah and Kouchakzadeh > Monzon *et al* > Churchill (1973)> Jain > Round > Manadilli > Evangleids *et al* > Avci Kargoz > Tsal > Churchill (1977) > Chen (1985). It is concluded that Newton Raphson ; Prandtl and Nikurdse; Zingrang and Sylvester ; Serghide ; Barr; Swamee and Jain; Eck ; Haaland ; Brkic ; Wood and Moody are first choice friction formulae based on the values of model of selection criterion.

Keywords: Darcy Friction Factor, Pipe Flow, Statistical Methods, Darcy Friction Formulae.

INTRODUCTION

Computation of flows, headloss and pressures in pipes have been of great value and interest for those involved in design, construction and maintenance of public water distribution systems. Many methods have been used in the past to compute flows, pressure and headloss in network of pipes. These methods range from graphical methods to the use of physical analogies and finally to the use of mathematical models (Lindell, 2006). Lindell (2006) gave the history of pipe network analysis and a summary of some of the more important methods as follows: Hardy Cross Method; The Simultaneous Node Method; The Simultaneous Loop Method; The Linear Method (Simultaneous Pipe Method) and The Gradient Method (Simultaneous Network Method). The most widely used method is the Hardy Cross method of balancing the heads at each of the junctions. It is an iteration procedure based on continuity as follows (Oke, 2007):

- The law of conservation of mass; and
- The pressures drop around any closed circuit is zero (France, 1993; Babatola *et al.,* 2008).

The procedure suggested by Hardy Cross requires that the flow in each pipe be initially estimated and the principle of continuity is satisfied at each junction. The head loss for each pipe in the network is determined from Hazen-Williams or Darcy-Weisbach (DW) equations. The most commonly used equation for obtaining the head loss in a pipe is the Darcy–Weisbach equation which is given by:

$$h_l = \frac{\lambda L V^2}{2gD} \text{ or } h_l = \frac{16\lambda L Q^2}{2g\pi^2 D^5} \quad (1)$$

Where; h_l is the head loss (m); is the friction factor (dimension less); L is the length of the pipe (m); D is the diameter of the pipe (m); V is mean velocity in the pipe (ms^{-1}) ; g is acceleration due to gravity (ms^{-2}) and Q is the discharge in the pipe

(m^3s^{-1}).

The accuracy of the DW depends on the heads loss; friction in a steady uniform flow and the corresponding discharge in each pipe. A correction to the assumed flow is computed successively for each loop in the network until the correction is reduced to an acceptable magnitude. The correction for each the loop is given by the equation:

$$\Delta Q = \frac{-\sum h_i}{2\sum\left(\frac{h_i}{Q_i}\right)} \quad (2)$$

Where; ΔQ is change in discharge in the pipe (m^3s^{-1}), Q_i is the discharge in the pipe (m^3s^{-1}) and h is as defined in equation 1.

The non-dimensional Darcy friction factor is a function of roughness coefficient (k), diameter of the pipe, and Reynolds number (R_e). Estimation of the Darcy friction factor is very important for the analysis of fluid mechanic behaviour in the pipes and open channels. The Darcy friction factor in turbulent flow of pipe depends on Reynolds number and relative roughness. There is an implicit relationship between Reynolds numbers and relative roughness

$$R_e = \frac{D}{\upsilon}V \quad (3)$$

Where; υ is kinematics viscosity of the liquid; R_e is Reynolds number and D and *V* are as defined in equation 1.

For laminar flow, Darcy friction factor (λ) is only dependent on the Reynolds number and is given by Moody, 1944 equation:

$$\lambda = \frac{64}{R_e} \quad (4)$$

But for the transitional region, is dependent on both Reynolds number and relative roughness. Darcy friction factor in a pipe can be computed by using implicit or explicit expressions (formulae) such as Colebrook-White's, Newton-Raphson, Moody, Wood, Barr and Haaland formulae. Colebrook–White (1937) and Colebrook (1939) expressions are given by (Mahendra, 2008; Mehran and Ayub, 2011) as:

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k}{3.7D} + \frac{2.51}{R_e\sqrt{\lambda}}\right) \quad (5a)$$

$$\frac{1}{\sqrt{\lambda}} = 1.74 - 2\log_{10}\left(2\frac{k}{D} + \frac{18.7}{R_e\sqrt{\lambda}}\right) \quad (5b)$$

Where; k is the effective roughness size of the pipe wall.

$$\frac{1}{\sqrt{\lambda}} = 1.14 - 2\log_{10}\left(1 + \frac{9.3}{R_e\frac{k}{D}\sqrt{\lambda}}\right) + 2\log\left(\frac{D}{k}\right) \quad (5c)$$

The Newton-Raphson method, which provides fast convergence of λ, is defined by Featherstone and Nalluri, 1982 as:

$$f(\lambda) = \frac{1}{\sqrt{\lambda}} + 2.\log_{10}\left(\frac{k}{3.7D} + \frac{2.51\pi Dv}{4Q\sqrt{\lambda}}\right) \quad (6)$$

The problem then reduces to finding the root of the function f(λ) = 0. The Newton–Raphson method requires a starting value (λ_o), that is the initial guess for λ and a first better approximation is given by France, 1993 as:

$$\lambda_{n+1} = \lambda_n - \left(\frac{f(\lambda_n)}{f^i(\lambda_n)}\right) \quad (7)$$

Where $f'(\lambda_n)$ is the first order derivative of $f(\lambda_n)$ at $\lambda_n = 0$.

Moody (1944; 1947) obtained directly without having to use a chart and proposed the expression for the range of $4 \times 10^3 \leq R_e \; 10^8$ and $0<k/D<5 \times 10^{-2}$. Moody expression is given by equation (8) (Moody, 1944; Babatola *et al.,* 2008).

$$\lambda = 0.0055\left(1 + \left[20000\frac{k}{D} + \frac{10^6}{R_e}\right]^{\frac{1}{3}}\right) \quad (8)$$

Wood (1966) recommended the following expressions for frictional factor (France, 1993)

$$\lambda = a + b\mathrm{Re}^{-c} \quad (9)$$

Where; a,b and c are Wood constants as follows:

$$a = 0.094\left(\frac{k}{D}\right)^{0.225} + 0.53\left(\frac{k}{D}\right) \quad (10)$$

$$b = 88.0\left(\frac{k}{D}\right)^{0.44} \quad (11)$$

and

$$c = 1.62\left(\frac{k}{D}\right)^{0.134} \quad (12)$$

Barr (1972; 1981) suggested an expression of the form:

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k}{3.7D} + \frac{5.158\log\left(\frac{R_e}{7}\right)}{R_e\left(1+\frac{R_e^{0.52}}{29}\frac{k}{D}\right)^{0.70}}\right) \quad (13)$$

In the last three decades, several explicit formulae have been suggested among which is Haaland (1983). Haaland suggested that:

$$\frac{1}{\sqrt{\lambda}} = -1.8\log_{10}\left(\frac{6.9}{R_e} + \left(\frac{k}{3.7D}\right)^{1.11}\right) \quad (14)$$

The work by Prandtl and Nikurdse (1982) on smooth and artificially roughness pipes revealed equations (15 and 16) for smooth, turbulent and rough turbulent respectively.

$$\frac{1}{\sqrt{\lambda}} = 2\log_{10}\left(\frac{R_e\sqrt{\lambda}}{2.51}\right) \quad (15)$$

$$\frac{1}{\sqrt{\lambda}} = 2\log_{10}\left(\frac{3.7D}{k}\right) \quad (16)$$

The Swamee and Jain (1976) equation below is used to solve directly for the Darcy-Weisbach Darcy friction factor for a full-flowing circular pipe.

$$\lambda = \left[\frac{0.25}{\left(\log_{10}\left(\frac{k}{3.7D} + \frac{5.74}{(R_e)^{0.9}}\right)\right)^2}\right] \text{ or}$$

$$\sqrt{\lambda} = -2\log_{10}\left(\frac{k}{3.7D} + \frac{5.74}{(R_e)^{0.9}}\right) \quad (17)$$

Like Swamee and Jain equation; Serghide (1984), Tsal(1989), and Zigrang and Sylester(1982) equations are used to determine the Darcy-Weisbach Darcy friction factor for a full-flowing circular pipe. Serghide (1984), Tsal(1989) , and Zigrang and Sylester (1982) equations are as follows (Mahendra, 2008):

Serghide (1984):

$$\lambda = \left[A - \frac{(B_s - A_s)^2}{(C_s - 2B_s + A_s)}\right]^{-2} \quad (18)$$

$$A_s = -2\log_{10}\left(\frac{k}{3.7D} + \frac{12}{R_e}\right) \quad (18a)$$

$$B_s = -2\log_{10}\left(\frac{k}{3.7D} + \frac{2.51A}{R_e}\right) \quad (18b)$$

$$C_s = -2\log_{10}\left(\frac{k}{3.7D} + \frac{2.51B}{R_e}\right) \quad (18c)$$

Tsal(1989):

$$\lambda = 0.11\left(\frac{k}{D} + \frac{68}{R_e}\right)^{0.25} \quad (19)$$

If $\lambda < 0018$ *then* $\lambda = 0.85\lambda + 0.0028$

Zigrang and Sylester(1982):

$$\lambda = \left[-2\log_{10}\left(\frac{k}{3.77D} - \frac{5.02}{R_e}\left(\log_{10}\left[\frac{k}{3.77D} - \frac{5.02}{R_e}\left\{\log_{10}\left(\frac{k}{3.77D} + \frac{13}{R_e}\right)\right\}\right]\right)\right)\right]^{-2} \quad (20)$$

Özger and Yıldırım (2009a and b) stated that the other widely used explicit expressions for Darcy friction factor determination are Churchill (1973; 1977), Jain (1976), Chen (1979; 1985), Manadilli (1997), Romeo *et al.* (2002); Sonnad and Goudar (2004; 2006). Equations (21 to 27) present these explicit expressions respectively.

Churchill (1973):

$$\frac{1}{\sqrt{\lambda}} = -2\left(\log_{10}\left(\left(\frac{k}{3.71D}\right) + \left(\frac{7}{(R_e)}\right)^{0.9}\right)\right) \quad (21)$$

Churchill (1977):

$$\lambda = 8\left[\left(\frac{8}{R_e}\right)^{12} + \frac{1}{(B_i + A_i)^{\frac{3}{2}}}\right]^{\frac{1}{12}} \quad (22)$$

$$A_i = \left(-2.457\ln\left(\left(\frac{k}{3.7D}\right) + \left(\frac{7}{R_e}\right)^{0.9}\right)\right)^{16} \quad (22a)$$

$$B_i = \left(\frac{37530}{R_e}\right)^{16} \quad (22b)$$

Jain (1976):

$$\frac{1}{\sqrt{\lambda}} = -2\left(\log_{10}\left(\left(\frac{k}{3.715D}\right) + \left(\frac{6.943}{R_e}\right)^{0.9}\right)\right) \quad (23)$$

Chen (1979):

$$\frac{1}{\sqrt{\lambda}} = -2\left(\log_{10}\left(\left(\frac{k}{3.7065D}\right) - \left(\frac{5.0452}{R_e}\right)\log_{10}\left(\left(\frac{1}{2.8257}\right)\left(\frac{k}{D}\right)^{1.1098} + \left(\frac{5.8506}{(R_e)^{0.8981}}\right)\right)\right)\right) \quad (24a)$$

Chen (1985):

$$\frac{1}{\sqrt{\lambda}} = -2\left(\log_{10}\left(\left(\frac{k}{3.7D}\right) - \left(\frac{95}{R_e^{0.983}}\right) - \left(\frac{96.82}{R_e}\right)\right)\right) \quad (24b)$$

Manadilli (1997):

$$\frac{1}{\sqrt{\lambda}} = -2\left(\log_{10}\left(\left(\frac{k}{3.7D}\right) + \left(\frac{95}{(R_e)^{0.983}}\right) - \left(\frac{96.82}{R_e}\right)\right)\right) \quad (25)$$

Romeo *et al.* (2002):

$$\frac{1}{\sqrt{\lambda}} = -2\left(\log_{10}\left(\left(\frac{k}{3.7065D}\right)-\left(\frac{5.0272}{(R_e)}\right)\right)\log_{10}\left(\frac{k}{3.827D}\right)-\left(\frac{4.567}{(R_e)}\right)* \log_{10}\left(\left(\frac{k}{7.7918D}\right)^{0.9924} - \left(\left(\frac{5.3326}{(208.815+R_e)}\right)^{0.9345}\right)\right)\right) \tag{26}$$

Sonnad and Goudar (2004; 2006)

$$\frac{1}{\sqrt{\lambda}} = 0.8686\ln\left(\frac{0.4587R_e}{S^{\frac{s}{s+1}}}\right) \text{ or}$$

$$\frac{1}{\sqrt{\lambda}} = 0.8686\ln\left(\frac{0.4587R_e}{(s-0.31)^{\frac{s}{s+1}}}\right) \tag{27}$$

$$s = 0.1240R_e\left(\frac{k}{D}\right) + \ln(0.4587R_e) \tag{27a}$$

Other Darcy friction factor formulae are Eck (1973); Round (1980); Vatankhah and Kouchakzadeh (2008); Buzzelli(2008); Avci and Kargoz(2009); Evanleids *et al.* (2010), Brkic (2011; 2012), which can be expressed as follows (Wikipedia, 2014):

Eck (1973):

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\left(\frac{k}{3.715D}\right)+\left(\frac{15}{R_e}\right)\right) \tag{28}$$

Round (1980):

$$\frac{1}{\sqrt{\lambda}} = 1.8\log\left(\frac{R_e}{0.135R_e\frac{k}{D}+6.5}\right) \tag{29}$$

Vatankhah and Kouchakzadeh (2008):

$$\frac{1}{\sqrt{\lambda}} = 0.8686\ln\left(\frac{0.4587R_e}{(s-0.31)^{\frac{s}{s+0.9633}}}\right) \tag{30}$$

$$s = 0.1240R_e\left(\frac{k}{D}\right) + \ln(0.4587R_e) \tag{30a}$$

Buzzelli (2008):

$$\frac{1}{\sqrt{\lambda}} = A_b - \left(\frac{A_b + 2\log\left(\frac{B_b}{R_e}\right)}{1+\frac{2.18}{B_b}}\right) \tag{31}$$

$$A_b = \left(\frac{0.744\ln(R_e)-1.41}{\left(1+1.32\sqrt{\frac{k}{D}}\right)}\right) \tag{31a}$$

$$B_b = \frac{k}{3.7D}R_e + 2.51A_b \tag{31b}$$

Avci and Kargoz(2009):

$$\lambda = \left(\frac{64}{\left(\ln(R_e)-\ln\left(1+0.001R_e\frac{k}{D}\left(1+10\sqrt{\frac{k}{D}}\right)\right)\right)^{2.4}}\right) \tag{32}$$

Evanleids *et al.* (2010):

$$\lambda = \left(\frac{0.2479-0.0000947(7-\log R_e)^4}{\left(\log\left(\frac{k}{3.615D}+\frac{7.366}{R_e^{0.9142}}\right)\right)^2}\right) \tag{33}$$

Brkic (2011; 2012)

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\left(\frac{k}{3.71D}\right)+\left(\frac{2.18S_b}{R_e}\right)\right) \tag{34}$$

$$S_b = \ln\left(\left(\frac{R_e}{1.816\ln\left(\frac{1.1R_e}{\ln(1+1.1R_e)}\right)}\right)\right) \tag{34a}$$

More on Darcy friction factor can be found in literature such as Mckeon *et al.* (2004; 2005) ; Ozger and Yildrim (2009a and b); Yildrim (2009); Xiande *et al.* (2011); Samadiafrad (2012); Salmasi *et al.* (2012); Coban (2012); Cojbasica and Brikic (2013); Papaevangelou *et al.* (2010); Winning and Cole (2013); Wikipedia (2014). With the well known importance of Darcy frictional factors in the selection of pipe size, determination of flows and headloss in a pipe, as well as important ingredient in the design of potable water supply scheme, coupled with alot of researches and publications on Darcy friction factors in pipe documentations on update of Darcy friction factor estimate are rare in literature in developing countries. With advancement in technology and development of high speed computer and internet there is the need to document update on Darcy friction factor estimate and provide performance evaluation of each of these Darcy friction factor formulae. The main aim of this study is to provide update on Darcy friction factor estimate and performance evaluation of each of these Darcy friction factor formulae with a particular attention to accuracy using statistical techniques.

MATERIALS AND METHOD

Darcy friction factor formulae for pipe flow problems were obtained from archives (Babatola *et al.,*2008; Mahendra, 2008; France, 1993; Featherstone and Nalluri, 1982; Mehran and Ayub, 2011; Winning and Coole, 2013; Wikipedia, 2014). These equations were used to estimate Darcy friction factor with Reynold number of $4 \times 10^3 \leq R_e$ 10^8 and relative roughness of $5 \times 10^{-5} < k/D < 0.01$. The results were evaluated statistically using total error, mean squared error; root squared error; model of selection (MSC); absolute error; mean error and root error with Colebrook White equation as the reference formula. Colebrook-White equation was used as the reference formulae because the formula is widely recommended as the formula to be used to estimate pipeline head loss (the roughness coefficient is a function of the surface roughness and independent of the size of the pipe or velocity of flow); it is applicable to a wide range of Reynolds number-from smooth to turbulent conditions and it provides slightly improved prediction of the wave velocity in pipe and the maximum depth of flow along the pipe.

RESULTS AND DISCUSSION

The results of the study were divided into two categories: estimated Darcy friction factors using other equations, and statistical evaluations

Estimated Darcy Friction Factors Using Other Equations: Tables 1 and 2 present values of estimated Darcy friction factors using all these Darcy friction factor formulae (techniques). From Table 1 the lowest Darcy friction factors at a fixed pipe diameter, Reynolds number and relative roughness came from Tsal. Colebrook–White equations, Haaland, Zigrang and Sylvester, Serghide, Swamee and Jain, Prandtl and Nikurdse, Moody and Newton Raphson equations gave moderate Darcy friction factors. The highest values of Darcy friction factor were obtained from Round, Wood and Barr equations. Figure 1 presents Moody chart. This indicates that the lower extreme equations (Darcy friction factor) should be avoided. From the figure (Figure 1) these Darcy friction factors can be grouped into two lower (painted red) and medium (painted blue) regions. This suggests that the two extreme equations (lowest and highest formulae) should be avoided.

Table 1: Estimated Darcy friction factor

Reynolds Number	k/D	Churchill (1973)	Jain	Chen (1979)	Round	AvciKargoz	Buzzelli	Sonnad and Goudar	Manadilli	Evangleids et al.	Vatankhah and Kouchakzadeh	Monzon et al.	Colebrook -White	Newton -Raphson
4000	0.000010	0.0406	0.0405	0.0398	0.0406	0.0399	0.0399	0.0398	0.0399	0.0400	0.0399	0.0399	0.0161	0.0157
4000	0.000910	0.0334	0.0333	0.0331	0.0334	0.0320	0.0331	0.0330	0.0332	0.0331	0.0331	0.0331	0.0161	0.0157
9000	0.005910	0.0337	0.0337	0.0334	0.0337	0.0226	0.0334	0.0333	0.0337	0.0335	0.0334	0.0334	0.0161	0.0157
59000	0.010910	0.0398	0.0398	0.0396	0.0398	0.0234	0.0396	0.0395	0.0398	0.0396	0.0396	0.0396	0.0201	0.0200
159000	0.020910	0.0497	0.0497	0.0496	0.0497	0.0268	0.0497	0.0497	0.0498	0.0497	0.0497	0.0497	0.0390	0.0391
209000	0.025910	0.0540	0.0540	0.0539	0.0540	0.0285	0.0540	0.0540	0.0541	0.0540	0.0540	0.0540	0.0446	0.0446
259000	0.030910	0.0580	0.0580	0.0579	0.0580	0.0301	0.0580	0.0580	0.0581	0.0581	0.0580	0.0580	0.0494	0.0495
309000	0.035910	0.0618	0.0618	0.0617	0.0618	0.0316	0.0618	0.0618	0.0619	0.0619	0.0618	0.0618	0.0538	0.0539
359000	0.040910	0.0654	0.0654	0.0653	0.0654	0.0330	0.0654	0.0654	0.0655	0.0655	0.0654	0.0654	0.0578	0.0579
909000	0.014100	0.0428	0.0428	0.0427	0.0428	0.0234	0.0428	0.0428	0.0428	0.0428	0.0428	0.0428	0.0652	0.0653
1409000	0.019100	0.0478	0.0478	0.0478	0.0478	0.0256	0.0478	0.0478	0.0478	0.0479	0.0478	0.0478	0.0367	0.0367
1909000	0.024100	0.0523	0.0523	0.0523	0.0523	0.0275	0.0523	0.0523	0.0523	0.0524	0.0523	0.0523	0.0427	0.0427
2409000	0.029100	0.0564	0.0564	0.0564	0.0564	0.0292	0.0565	0.0565	0.0565	0.0565	0.0565	0.0565	0.0477	0.0478
2909000	0.034100	0.0603	0.0603	0.0603	0.0603	0.0308	0.0604	0.0604	0.0604	0.0605	0.0604	0.0604	0.0522	0.0523
3409000	0.039100	0.0640	0.0640	0.0640	0.0640	0.0323	0.0640	0.0640	0.0640	0.0642	0.0640	0.0640	0.0564	0.0565
3909000	0.044100	0.0675	0.0675	0.0675	0.0675	0.0337	0.0676	0.0676	0.0676	0.0677	0.0676	0.0676	0.0602	0.0603
4409000	0.049100	0.0709	0.0709	0.0709	0.0709	0.0351	0.0710	0.0710	0.0710	0.0711	0.0710	0.0710	0.0639	0.0640
9909000	0.010419	0.0384	0.0384	0.0384	0.0384	0.0214	0.0384	0.0384	0.0384	0.0384	0.0384	0.0384	0.0708	0.0710
10409000	0.010919	0.0390	0.0390	0.0390	0.0390	0.0216	0.0391	0.0391	0.0391	0.0391	0.0391	0.0391	0.0377	0.0378
10909000	0.011419	0.0396	0.0396	0.0396	0.0396	0.0219	0.0397	0.0397	0.0397	0.0397	0.0397	0.0397	0.0384	0.0384
11409000	0.011919	0.0402	0.0402	0.0402	0.0402	0.0222	0.0403	0.0403	0.0403	0.0403	0.0403	0.0403	0.0390	0.0391
11909000	0.012419	0.0408	0.0408	0.0408	0.0408	0.0224	0.0408	0.0408	0.0409	0.0408	0.0408	0.0408	0.0396	0.0397
12409000	0.012919	0.0414	0.0414	0.0414	0.0414	0.0227	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0402	0.0403
12909000	0.013419	0.0419	0.0419	0.0420	0.0419	0.0229	0.0420	0.0420	0.0420	0.0420	0.0420	0.0420	0.0408	0.0408
13909000	0.014419	0.0430	0.0430	0.0430	0.0430	0.0234	0.0431	0.0431	0.0431	0.0431	0.0431	0.0431	0.0419	0.0420
14409000	0.014919	0.0436	0.0436	0.0436	0.0436	0.0237	0.0436	0.0436	0.0436	0.0436	0.0436	0.0436	0.0425	0.0425
14909000	0.015419	0.0441	0.0441	0.0441	0.0441	0.0239	0.0441	0.0441	0.0441	0.0441	0.0441	0.0441	0.0430	0.0431
15909000	0.016419	0.0451	0.0451	0.0451	0.0451	0.0243	0.0452	0.0452	0.0452	0.0452	0.0452	0.0452	0.0441	0.0441
16409000	0.016919	0.0456	0.0456	0.0456	0.0456	0.0246	0.0457	0.0457	0.0457	0.0457	0.0457	0.0457	0.0446	0.0446
16909000	0.006701	0.0332	0.0332	0.0332	0.0332	0.0189	0.0333	0.0333	0.0333	0.0332	0.0333	0.0333	0.0451	0.0452
66909000	0.011701	0.0400	0.0400	0.0400	0.0400	0.0220	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0456	0.0457
116909000	0.016701	0.0454	0.0454	0.0454	0.0454	0.0245	0.0454	0.0454	0.0454	0.0454	0.0454	0.0454	0.0332	0.0332
166909000	0.021701	0.0501	0.0501	0.0502	0.0501	0.0265	0.0502	0.0502	0.0502	0.0502	0.0502	0.0502	0.0399	0.0400
216909000	0.026701	0.0544	0.0544	0.0545	0.0544	0.0284	0.0545	0.0545	0.0545	0.0545	0.0545	0.0545	0.0454	0.0454
266909000	0.031701	0.0584	0.0584	0.0585	0.0584	0.0300	0.0585	0.0585	0.0585	0.0585	0.0585	0.0585	0.0501	0.0502
366909000	0.041701	0.0658	0.0658	0.0658	0.0658	0.0331	0.0659	0.0659	0.0659	0.0658	0.0659	0.0659	0.0584	0.0585
416909000	0.046701	0.0692	0.0692	0.0693	0.0692	0.0344	0.0693	0.0693	0.0693	0.0693	0.0693	0.0693	0.0622	0.0623
466909000	0.051701	0.0726	0.0726	0.0726	0.0726	0.0357	0.0727	0.0727	0.0727	0.0726	0.0727	0.0727	0.0658	0.0659
516909000	0.005679	0.0315	0.0315	0.0316	0.0315	0.0182	0.0316	0.0316	0.0316	0.0314	0.0316	0.0316	0.0692	0.0693
566909000	0.006179	0.0324	0.0324	0.0324	0.0324	0.0185	0.0324	0.0324	0.0324	0.0322	0.0324	0.0324	0.0725	0.0727
716909000	0.007679	0.0347	0.0347	0.0347	0.0347	0.0196	0.0347	0.0347	0.0347	0.0345	0.0347	0.0347	0.0332	0.0332
816909000	0.008679	0.0361	0.0361	0.0361	0.0361	0.0203	0.0362	0.0362	0.0362	0.0359	0.0362	0.0362	0.0347	0.0347
866909000	0.009179	0.0368	0.0368	0.0368	0.0368	0.0206	0.0368	0.0368	0.0368	0.0366	0.0368	0.0368	0.0354	0.0355

Table 1: Estimated Darcy friction factor

Reynolds Number	k/D	Moody	Wood	Barr	Prandtl and Nikurdse	Swamee and Jain	Serghide	Tsal	Zigrang and Sylvester	Haaland	Brkic	Eck	Churchil (1977)	Chen (1985)
4000	0.000910	0.0402	0.0385	0.0401	0.0081	0.0405	0.0399	0.0069	0.0080	0.0400	0.0442	0.0425	0.0406	0.0399
9000	0.005910	0.0402	0.0385	0.0401	0.0081	0.0405	0.0399	0.0069	0.0080	0.0400	0.0442	0.0425	0.0411	0.0404
59000	0.010910	0.0333	0.0325	0.0332	0.0192	0.0333	0.0331	0.0043	0.0191	0.0325	0.0359	0.0338	0.0334	0.0332
159000	0.020910	0.0391	0.0402	0.0398	0.0390	0.0398	0.0396	0.0053	0.0388	0.0392	0.0400	0.0396	0.0398	0.0398
209000	0.025910	0.0433	0.0457	0.0451	0.0446	0.0451	0.0449	0.0067	0.0443	0.0445	0.0452	0.0450	0.0451	0.0451
259000	0.030910	0.0468	0.0506	0.0498	0.0495	0.0498	0.0497	0.0080	0.0491	0.0492	0.0498	0.0497	0.0498	0.0498
309000	0.035910	0.0498	0.0551	0.0541	0.0538	0.0541	0.0540	0.0092	0.0534	0.0535	0.0541	0.0540	0.0541	0.0541
359000	0.040910	0.0524	0.0594	0.0581	0.0579	0.0581	0.0580	0.0104	0.0574	0.0575	0.0581	0.0580	0.0580	0.0581
909000	0.014100	0.0570	0.0675	0.0655	0.0653	0.0655	0.0654	0.0125	0.0648	0.0648	0.0654	0.0653	0.0290	0.0290
1409000	0.019100	0.0367	0.0376	0.0368	0.0367	0.0368	0.0368	0.0045	0.0365	0.0365	0.0368	0.0368	0.0368	0.0368
1909000	0.024100	0.0416	0.0435	0.0428	0.0427	0.0428	0.0428	0.0061	0.0424	0.0424	0.0428	0.0427	0.0428	0.0428
2409000	0.029100	0.0454	0.0487	0.0478	0.0478	0.0478	0.0478	0.0075	0.0475	0.0474	0.0478	0.0478	0.0478	0.0478
2909000	0.034100	0.0486	0.0534	0.0523	0.0523	0.0523	0.0523	0.0087	0.0519	0.0518	0.0523	0.0523	0.0523	0.0523
3409000	0.039100	0.0514	0.0578	0.0565	0.0565	0.0565	0.0565	0.0099	0.0560	0.0560	0.0564	0.0564	0.0564	0.0565
3909000	0.044100	0.0539	0.0620	0.0604	0.0603	0.0604	0.0604	0.0111	0.0599	0.0598	0.0603	0.0603	0.0603	0.0604
4409000	0.049100	0.0562	0.0661	0.0640	0.0640	0.0640	0.0640	0.0121	0.0635	0.0635	0.0640	0.0640	0.0640	0.0640
9909000	0.010419	0.0602	0.0737	0.0710	0.0710	0.0710	0.0710	0.0142	0.0703	0.0703	0.0709	0.0709	0.0302	0.0303
10409000	0.010919	0.0376	0.0386	0.0378	0.0378	0.0378	0.0378	0.0048	0.0376	0.0375	0.0378	0.0378	0.0378	0.0378
10909000	0.011419	0.0381	0.0392	0.0384	0.0384	0.0384	0.0384	0.0049	0.0382	0.0381	0.0384	0.0384	0.0384	0.0384
11409000	0.011919	0.0386	0.0398	0.0391	0.0391	0.0391	0.0391	0.0051	0.0388	0.0387	0.0390	0.0390	0.0390	0.0391
11909000	0.012419	0.0391	0.0404	0.0397	0.0397	0.0397	0.0397	0.0053	0.0394	0.0393	0.0396	0.0396	0.0397	0.0397
12409000	0.012919	0.0396	0.0410	0.0403	0.0403	0.0403	0.0403	0.0054	0.0400	0.0399	0.0402	0.0402	0.0402	0.0403
12909000	0.013419	0.0401	0.0416	0.0409	0.0408	0.0409	0.0408	0.0056	0.0406	0.0405	0.0408	0.0408	0.0408	0.0409
13909000	0.014419	0.0410	0.0428	0.0420	0.0420	0.0420	0.0420	0.0059	0.0417	0.0416	0.0419	0.0419	0.0420	0.0420
14409000	0.014919	0.0414	0.0433	0.0425	0.0425	0.0425	0.0425	0.0060	0.0422	0.0421	0.0425	0.0425	0.0425	0.0425
14909000	0.015419	0.0418	0.0439	0.0431	0.0431	0.0431	0.0431	0.0062	0.0428	0.0427	0.0430	0.0430	0.0431	0.0431
15909000	0.016419	0.0427	0.0449	0.0441	0.0441	0.0441	0.0441	0.0064	0.0438	0.0437	0.0441	0.0441	0.0441	0.0441
16409000	0.016919	0.0431	0.0455	0.0447	0.0446	0.0447	0.0447	0.0066	0.0443	0.0442	0.0446	0.0446	0.0446	0.0447
16909000	0.006701	0.0434	0.0460	0.0452	0.0452	0.0452	0.0452	0.0067	0.0448	0.0448	0.0451	0.0451	0.0451	0.0452
66909000	0.011701	0.0438	0.0465	0.0457	0.0457	0.0457	0.0457	0.0069	0.0453	0.0453	0.0456	0.0456	0.0225	0.0225
116909000	0.016701	0.0336	0.0340	0.0333	0.0332	0.0333	0.0333	0.0037	0.0331	0.0329	0.0332	0.0332	0.0332	0.0333
166909000	0.021701	0.0394	0.0408	0.0400	0.0400	0.0400	0.0400	0.0053	0.0397	0.0396	0.0400	0.0400	0.0400	0.0400
216909000	0.026701	0.0437	0.0463	0.0454	0.0454	0.0454	0.0454	0.0068	0.0451	0.0450	0.0454	0.0454	0.0454	0.0454
266909000	0.031701	0.0471	0.0512	0.0502	0.0502	0.0502	0.0502	0.0081	0.0498	0.0497	0.0501	0.0501	0.0502	0.0502
316909000	0.036701	0.0501	0.0558	0.0545	0.0545	0.0545	0.0545	0.0094	0.0541	0.0540	0.0544	0.0544	0.0545	0.0545
366909000	0.041701	0.0527	0.0600	0.0585	0.0585	0.0585	0.0585	0.0105	0.0580	0.0580	0.0584	0.0584	0.0585	0.0585
416909000	0.046701	0.0551	0.0641	0.0623	0.0623	0.0623	0.0623	0.0116	0.0618	0.0617	0.0622	0.0622	0.0622	0.0623
466909000	0.051701	0.0573	0.0681	0.0659	0.0659	0.0659	0.0659	0.0127	0.0653	0.0653	0.0658	0.0658	0.0658	0.0659
516909000	0.005679	0.0593	0.0719	0.0693	0.0693	0.0693	0.0693	0.0137	0.0687	0.0687	0.0692	0.0692	0.0693	0.0693
566909000	0.006179	0.0611	0.0757	0.0727	0.0727	0.0727	0.0727	0.0147	0.0720	0.0720	0.0726	0.0726	0.0307	0.0307
716909000	0.007679	0.0336	0.0340	0.0332	0.0332	0.0332	0.0332	0.0036	0.0330	0.0329	0.0332	0.0332	0.0332	0.0332
816909000	0.008679	0.0350	0.0355	0.0347	0.0347	0.0347	0.0347	0.0040	0.0345	0.0344	0.0347	0.0347	0.0347	0.0347
866909000	0.009179	0.0356	0.0362	0.0355	0.0355	0.0355	0.0355	0.0042	0.0352	0.0351	0.0354	0.0354	0.0354	0.0355

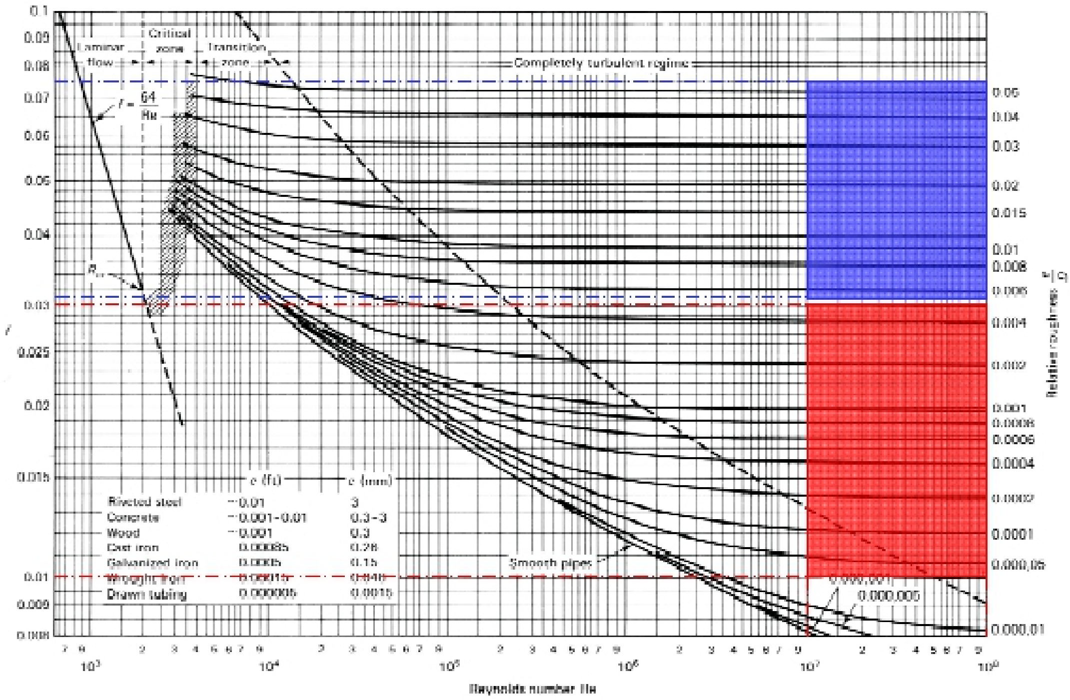

Figure 1: Moody Chart for Darcy Friction Factor Estimate (Source: Moody, 1944; Babatola *et al.,* 2008)

Statistical Evaluations: The model of selection criterion (MSC) is interpreted as the proportion of expected data variation that can be explained by the obtained data. It is well known that the higher the value of MSC, the higher is the accuracy, validity and the good fitness of the method. MSC can be computed using equation (35) (Oke, 2007) as follows :

$$MSC = \ln \frac{\sum_{i=1}^{n}\left(Y_{obsi} - \overline{Y_{obs}}\right)^2}{\sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2} - \frac{2p}{n} \qquad (35)$$

Where; MSC is the model of selection criterion; n is the number of samples; p is the number of parameters for MSC; $\overline{Y_{obs}}$ is the mean calculated Darcy friction factor for the Colebrook-White model; Y_{cali} is the calculated Darcy friction factor for the models and Y_{obsi} is the calculated Darcy friction factor for the Colebrook-White model

Table 2 presents the values of MSC for these equations. The MSC values are 9.598; 0.452; 1.831; 1.662; 4.381; 1.630; 1.683; -5.050; 4.325; 1.668; -0.42; - 0.040; -0.025; -0.042; -2.946; -0.025; -0.022; -0.023;; -0.029; -0.024; -0.024; 1.312; 1.476; -0.036 and -0.025 for Newton Raphson, Moody, Wood, Barr, Prandtl and Nikurdse, Swamee and Jain , Serghide, Tsal, Zigrang and Sylvester, Haaland; Churchill (1979); Jain; Chen (1983); Round; AvciKargoz; Buzzelli; Sonnad and Goudar; Manadilli; Evangleids *et al.*; Vatankhah and Kouchakzadeh; Monzon *et al.*; Brkic; Eck; Churchil (1977) and Chen (1985) respectively. These MSC values revealed that Darcy friction factor formulae can be grouped into three. Formulae with friction factor lower than Colebrook –White (MSC value less than zero), formulae with friction factor higher than Colebrook –White (MSC value greater than 1.5) and formulae with friction factor closer to Colebrook –White (MSC value greater than zero but less than 1.5).

Total Error: The lower the value of total error, the higher is the accuracy, validity and good fitness of the method. Total error (Err^2) can be computed using equation (36):

$$Err^2 = \sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2 \qquad (36)$$

Where; Err^2 is total error ; Y_{obsi} is observed value using Colebrook–White and Y_{cali} is calculated value using formular. Table 2 presents the values of total error for these equations.

Absolute Error: The lower the value of absolute error the higher is the accuracy, validity and good fitness of the method. Absolute error (AbErr) can be computed using equation (37):

$$AbErr = \sum_{i=1}^{n} \left|\left(Y_{obsi} - Y_{cali}\right)\right| \quad (37)$$

Where; AbErr is absolute error. Table 2 presents the values of absolute error for these equations.

Mean Error: The lower the value of mean error the higher is the accuracy, validity and good fitness of the method. Mean error (MnErr) can be computed using equation (38):

$$MnErr = \frac{\sum_{i=1}^{n} \left|\left(Y_{obsi} - Y_{cali}\right)\right|}{n} \quad (38)$$

Table 2: Statistical Evaluation of Darcy Friction Factor Formulae

Darcy friction factor formulae	Model of selection criterion	Total error	Mean Squared error	Root squared error	Absolute error	Mean error	Root error
Churchill	-0.042	0.007	0.000	0.082	0.210	0.004	0.458
Jain	-0.040	0.007	0.000	0.082	0.210	0.004	0.458
Chen	-0.025	0.007	0.000	0.081	0.208	0.004	0.456
Round	-0.042	0.007	0.000	0.082	0.210	0.004	0.458
Avci Kargoz	-2.946	0.033	0.001	0.180	1.228	0.022	1.108
Buzzelli	-0.025	0.007	0.000	0.081	0.209	0.004	0.457
Sonnad and Goudar	-0.022	0.007	0.000	0.081	0.209	0.004	0.457
Manadilli	-0.023	0.007	0.000	0.081	0.211	0.004	0.459
Evangleids et al	-0.029	0.007	0.000	0.081	0.211	0.004	0.459
Vatankhah and Kouchakzadeh	-0.024	0.007	0.000	0.081	0.209	0.004	0.457
Monzon et al	-0.024	0.007	0.000	0.081	0.209	0.004	0.457
Newton - Raphson	9.598	0.000	0.000	0.001	0.005	0.000	0.068
Moody	0.452	0.002	0.000	0.048	0.207	0.004	0.454
Wood	1.831	0.001	0.000	0.035	0.122	0.002	0.349
Barr	1.662	0.001	0.000	0.037	0.069	0.001	0.264
Prandtl and Nikurdse	4.381	0.000	0.000	0.011	0.021	0.000	0.144
Swamee and Jain	1.630	0.001	0.000	0.037	0.070	0.001	0.265
Serghide	1.683	0.001	0.000	0.036	0.067	0.001	0.260
Tsal	-5.050	0.084	0.002	0.289	2.050	0.038	1.432
Zigrang and Sylvester	4.325	0.000	0.000	0.012	0.031	0.001	0.175
Haaland	1.668	0.001	0.000	0.036	0.078	0.001	0.279
Brkic	1.312	0.002	0.000	0.043	0.079	0.001	0.281
Eck	1.476	0.002	0.000	0.040	0.071	0.001	0.267
Churchill (1977)	-0.036	0.002	0.009	0.000	0.045	0.256	0.005
Chen (1985)	-0.025	0.002	0.009	0.000	0.045	0.256	0.005

Where; MnErr is mean error. Table 2 presents the values of error for these equations.

Root Squared Error: The lower the value of root squared error the higher the accuracy, validity and good fitness of the method. Root squared error (RSErr) can be computed using equation (39):

$$RSErr = \sqrt{\sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2} \qquad (39)$$

Where; RsErr is root squared error. Table 2 presents the values of error for these equations.

Root Absolute Error: The lower the value of root absolute error the higher the accuracy, validity and good fitness of the method. Root absolute error (RabErr) can be computed using equation (40):

$$RabErr = \sqrt{\sum_{i=1}^{n}\left|\left(Y_{obsi} - Y_{cali}\right)\right|} \qquad (40)$$

Where; RabErr is root squared error. Table 2 presents the values of error for these equations.

Mean Squared Error: The lower the value of mean squared error the higher the accuracy, validity and good fitness of the method. Mean squared error ($MnErr^2$) can be computed using equation (41):

$$MnErr^2 = \frac{\sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2}{N} \qquad (41)$$

Where; $MnErr^2$ is mean squared error. Table 2 presents the values of error for these equations.

Based on the values of these errors Newton Raphson > Prandtl and Nikurdse > Zingrang and Sylvester > Serghide > Barr > Swamee and Jain > Eck > Haaland > Brkic > Wood > Moody > Chen (1979) > Buzzelli > Sonnad and Goudar > Vatankhah and Kouchakzadeh > Monzon *et al.* > Churchill > Jain > Round > Manadilli > Evangleids *et al.* > Avci Kargoz > Tsal > Churchill (1977) > Chen (1985).
(Where; > is stands for greater than)

CONCLUSION

The study provides an update on Darcy friction factor and their performance evaluation. The study revealed that there are 25 Darcy friction factor formulae which depend on Reynolds number and relative roughness. It can be concluded that accuracy of the formulae are in order of:

i. Newton Raphson > Prandtl and Nikurdse > Zigrang and Sylvester > Wood > Serghide > Haaland > Barr > Swamee and Jain > Eck > Brkic > Moody > Sonnad and Goudar > Manadilli > Monzon *et al* > Vatankhah and Kouchakzadeh > Chen (1985) based the value model of selection criterion;

ii. Newton Raphson > Prandtl and Nikurdse > Zingrang and Sylvester > Serghide > Barr > Swamee and Jain > Eck > Haaland > Brkic > Wood > Moody > Chen (1979) > Buzzelli > Sonnad and Goudar > Vatankhah and Kouchakzadeh > Monzon *et al.* > Churchill > Jain > Round > Manadilli > Evangleids *et al.* > Avci Kargoz > Tsal > Churchill (1977) > Chen (1985) based on mean error; and

iii. Although, it might be argued that computation of friction factors using these higher accuracy formulae might be difficult, but availability of high speed computer and other devices have made it easier.

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