EXAMINING THE EFFICACY OF AN ITERATIVE MEASUREMENT ERROR ADJUSTMENT TECHNIQUE ON THE ADEQUACY OF A 2-LEVEL MODEL

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ABSTRACT

This paper proposes an iterative technique to realistically adjust for the incidence of measurement errors in multilevel models using a 2-level model framework. The technique yields the expected measurement error adjustment benefits but shows that such benefits do not necessarily accrue when all perceived error-prone predictor variables in a model are simultaneously adjusted for errors.

Keywords: Multilevel Model, Measurement Errors, Coefficient of Variation, Predictor Variable, Model Deviance.

INTRODUCTION

In many of the variables used in the physical, biological, social and medical science, measurement errors are found. These errors, which could be problematic in statistical inference, are essentially random or systematic. In fixed effects models such as linear and generalized linear models, there is a wealth of literature (e.g. Joreskog, 1970; Degracie and Fuller, 1972; Plewis, 1985; Fuller, 1987; Carroll *et al*, 1995; Skrondal and Rabe-Hesketh, 2004 and Fuller, 2006) elaborating the consequences of measurement errors on model adequacy especially in situations where such errors are apparent and non-ignorable. Some of the adverse consequences of not adjusting for the incidence of measurement errors include asymptotic bias of the error-prone predictor variable coefficient estimate, reduced predictive power, increased model deviance and reduced coefficient estimate standard error. The efficacy of mixed effects models such as multilevel (hierarchical) linear models is also adversely affected by a failure to properly account for measurement errors in their formulation and/or estimation. However, as pointed out by Goldstein *et al*. (2008), the behaviour of biases associated with measurement error in covariates or the response for multilevel linear models is, up to date, not well known and can be complex.

At the crux of measurement error adjustment approaches in the affected models, is the need to properly estimate the measurement error variances and reliabilities of the error-prone techniques for estimating measurement variances are, in general, deficient. There is inability to sufficiently justify independence of measurement errors and the so called unidimensionality assumption as required in educational mental testing ; accuracy and consistency of the estimates of the measurement error variance could not be guaranteed (Ecob and Goldstein, 1983). The method of instrumental variables strongly recommended for certain situations as in mental testing (see Ecob and Goldstein, 1983) requires, however, that several different instrumental variables be considered for comparison. There is also the difficulty of establishing that measurement errors are independent of instrumental variables (sargan, 1958).

In many applied or theoretical considerations, measurement error variance and the reliability of the associated error-prone variable is assumed known rather than estimated (Goldstein *et al*., 2008). The measurements errors associated with explanatory variables that cannot be observed directly are often ignored or the analyses are carried out using assumptions that may not always be realistic (Aitkin and Longford, 1986; Goldstein, 1987). This paper proposes the bootstrapping approach to realistically estimate measurement error variance of perceived error-prone predictors with random coefficients and shows how these estimates(rather than assumed values) can be validly used to adjust for the incidence of measurement errors in multilevel models and hence improve overall model

METHODOLOGY

Data Structure in the Study

The study utilized a 2-level data structure in which post primary school students constituted level 1 units while the schools constituted level 2 units. Average scores for selected subjects in Junior Secondary School 1 (JSS1), Junior Secondary School Certificate Examination (JSSCE) as well as Senior Secondary School Certificate Examination (SSCE) or West African School Certificate Examination (WASCE) for each student in a sample "statistical cohort" of students in any school were captured between 2002 and 2008. In the original collection (called Data 1), there were 1,111 level 1 units and 50 level 2 units. Using simulation, more data were generated to obtain two other scenarios: the one retaining same number of level 2 units but having 2,222 level 1 units (Data 2) while the other had 110 level 2 units and 4022 level 1 units (Data 3).

Description of Variables

Table 1 below summarizes a description of the variables used in the study.

Table 1: Variables used in the Study

Variable name	Description
Y_{ij}	Science, Technology and Mathematics (STM) score per student in all classes; a level 1 response variable
X_{1ij}	STM score per student in JSS1 subjects; a level 1 predictor variable
X_{2ij}	STM score per student in JSSCE subjects; a level 1 predictor variable
X_{3j}	Final School STM score; a level 2 predictor variable
X_{nj}	The school system; it is a categorical level 2 predictor variable (where n = 4, 5 or 6) with the systems categorized into "Boardsytem" ; X_{4j} , " Daysystem" ; X_{5j} or "Bothsystem" ; X_{6j} .
cons	Constant used for dummy variables and usually carries a value of one ; it is a level 1 predictor .

The observed variables X_{1ij}, X_{2ij} and X_{3j} are centred around appropriate means

The 2-level Model

In terms of the true unobservable forms of the variables described in Table 1 above, the 2-level model to be examined takes the form

$$y_{ij} = \beta_{0j} + \beta_{1j}x_{1ij} + \beta_{2j}x_{2ij} + \beta_3 x_{3j} + \beta_4 x_{4j} + \beta_5 x_{6j} + \beta_7 (x_1 \bullet x_4)_{ij} + \beta_8 (x_2 \bullet x_6)_{ij \cdot ij} + e_{ij}$$

with

$$\beta_{0j} = \beta_o + u_{0j}$$

$$\beta_{1j} = \beta_1 + u_{1j}$$

$$\beta_{2j} = \beta_2 + u_{2j}$$

and

$$\begin{bmatrix} u_{0j} \\ u_{1j} \\ u_{2j} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} \sigma^2_{uo} & & \\ \sigma_{uo1} & \sigma^2_{u1} & \\ \sigma_{uo2} & \sigma_{u12} & \sigma^2_{u2} \end{bmatrix}$$

$$e_{ij} \sim N(0, \sigma^2_e). \quad (2.1)$$

It is assumed that

$$\mathrm{cov}(u_j, u_{j'}) = \mathrm{cov}(e_{ij}, e_{i'j}) = 0,$$
$$\mathrm{cov}(u_j, e_{ij}) = 0,$$
$$E(u_j) = E(e_{ij}) = 0 \quad (2.2)$$

The parameters to be estimated are β_0, β_1, β_2, β_3, β_4, β_5, β_7, β_8, (the fixed parameters) and Ω_u and σ^2_e (the random parameters). It is intended to examine the effect of adjusting for the incidence of measurement errors perceived to be inherent in X_{1ij} and X_{2ij} on the parameter estimates and reliability of the response variable Y_{ij} (where the measurement error variance of Y_{ij} is assumed). Estimated measurement error variance values and hence reliabilities for X_{1ij} and X_{2ij} are, however, considered in the analysis alongside scenarios where measurement error variance values for X_{1ij} and X_{2ij} were also assumed.

The measurement error models of the predictor variables and response variable subject to error are:

$$X_{1ij} = x_{1ij} + m_{1ij},$$
$$X_{2ij} = x_{2ij} + m_{2ij},$$
$$Y_{ij} = y_{ij} + q_{ij}. \quad (2.3)$$

We assume that the errors m_{1ij} are distributed with zero mean and constant variance and that they are not correlated either with one another across pupils or with the true values x_{1ij}. We make similar assumptions about the errors m_{2ij} and q_{ij}. Now, where as the adjustment for incidence of errors in Y_{ij} were based on an assumed variance, var(q), and hence reliability of Y_{ij}, the cases of X_{1ij} and X_{2ij} employed variances of m_1 and m_2 that were estimated using an iterative technique elucidated in section 2.4.

The Iterative Measurement Error Adjustment Technique

The technique entails the following steps:

(I) From each group (or subgroup) of the multilevel model obtain an estimate of the explanatory variable mean, $\overline{X}_{\bullet j}$, based on sample sizes of at least 30 in each group.

(ii) Average these $\overline{X}_{\bullet j}$'s (using arithmetic mean) across the entire groups to obtain a value, say $\overline{X}$.

(iii) Estimate the measurement error (ME) variance, σ^2_{hm}, as the mean of the squares of deviations of $\overline{X}_{\bullet j}$'s from $\overline{X}$.

(iv) Estimate σ^2_{hX} as in the first paradigm approach and hence estimate R_h accordingly.

(v) Use the values σ^2_{hm} and σ^2_{hX} to adjust for measurement error in the variable (s) of interest and hence re-estimate the k-level model accordingly via Gibbs sampling in MCMC.

(vi) Check for possible attenuation and/or inconsistency of the estimated multilevel parameters.

(vii) If there is attenuation (reduced or no increase in predictive power of corresponding predictor) and/or inconsistency of the estimated multilevel parameters then repeat steps (i) to (vi) , possibly increasing re-sampling size per cluster and/or increasing number of samples.

ANALYSIS AND DISCUSSION

The iterative technique (2.4) was employed and measurement error variances and reliabilities of the variables X_{1ij} and X_{2ij} estimates were obtained. Table 2 reflects these estimates.

Table 2: Estimated Variances, Measurement Error (M.E) Variances and Reliabilities (based on data 1 and 2) in respect of the 'student's subject score per class' Predictor Variable (X_{1ij}) and STM score per student in JSSCE subjects (X_{2ij}).

Data	Variable	Variance	M.E Variance	Reliability
1	X_{1ij}	0.74	0.26	0.74
	X_{2ij}	0.53	0.47	0.53
2	X_{1ij}	0.67	0.25	0.73
	X_{2ij}	0.48	0.46	0.51

The measurement error variance estimates of X_{1ij} and X_{2ij} using data 3 were 0.64 and 0.82, respectively are an indication of highly error-prone values that were generated in the simulation of more level 2 units (and hence more level 1 units). These estimates differ remarkably from what obtained in data 1 and 2 scenarios. The study shall adopt the average of the estimates in data 1 and 2 (i.e 0.26 and 0.46) to examine measurement error adjustment effects for the model using the three data sets and then also examine the measurement error adjustment effect for the model using data 3 and considering the measurement error variances estimated only from the data 3 scenario.

The Tables 3-6 that now follow summarize some of the results associated with using M.E variances obtainable via the iterative technique described in 2.4.

Table 3 : Estimated coefficient estimates for X_{1ij} and X_{2ij} , coefficients of variation as well as model deviances and residual variances based on data 1 under varying M.E adjustments.

Parameters Estimated	Estimates Prior to M.E Adjustments	Error Adjusted estimates with M.E variance of $X_1 = 0.26$ and that of $X_2 = 0.46$	Error Adjusted estimates with M.E variance of $X_1 = 0.26$	Error Adjusted estimates with M.E variance of $X_2 = 0.46$
β_1	0.877(0.062)	0.311(0.075)	0.761(0.026)	0.855 (0.074)
β_2	0.352(0.016)	0.827(0.053)	0.366 (0.018)	0.874 (0.040)
Coefficient of variation (CV) for β_1	0.071	0.24	0.034	0.087
Coefficient of variation (CV) for β_2	0.045	0.064	0.049	0.046
Model Deviance	803	-3597	-3687	-4238
Residual variance	0.101	0.003	0.003	0.002

Table 4 : Estimated coefficient estimates for X_{1ij} and X_{2ij}, coefficients of variation as well as model deviances and residual variances based on data 2 under varying M.E adjustments.

Parameters Estimated	Estimates Prior to M.E Adjustments	Error Adjusted estimates with M.E variance of X_1 = 0.26 and that of X_2 = 0.46	Error Adjusted estimates with M.E variance of X_1 = 0.26	Error Adjusted estimates with M.E variance of X_2 = 0.46
β_1	0.819(0.066)	0.198(0.023)	0.706(0.044)	0.740(0.102)
β_2	0.348(0.018)	0.807(0.051)	0.359(0.014)	0.816(0.034)
Coefficient of variation (CV) for β_1	0.081	0.116	0.062	0.139
Coefficient of variation (CV) for β_2	0.052	0.063	0.039	0.042
Model Deviance	1187	-9272	-6824	-8852
Residual variance	0.086	0.001	0.003	0.001

Table 5 : Estimated coefficient estimates for X_{1ij} and X_{2ij} , coefficients of variation as well as model deviances and residual variances based on data 3 under varying M.E adjustments.

Parameters Estimated	Estimates Prior to M.E Adjustments	Error Adjusted estimates with M.E variance of X_1 = 0.26 and that of X_2 = 0.46	Error Adjusted estimates with M.E variance of X_1 = 0.26	Error Adjusted estimates with M.E variance of X_2 = 0.46
β_1	0.982(0.022)	0.426(0.021)	0.836(0.022)	0.916(0.022)
β_2	0.339 (0.011)	0.681(0.014)	0.369(0.011)	0.818(0.020)
Coefficient of variation (CV) for β_1	0.022	0.049	0.026	0.024
Coefficient of variation (CV) for β_2	0.032	0.021	0.030	0.024
Model Deviance	3366	-17659	-14799	-16022
Residual variance	0.113	0.001	0.002	0.001

Table 6 : Estimated coefficient estimates for X_{1ij} and X_{2ij} , coefficients of variation as well as model deviances and residual variances based on data 3 under varying M.E adjustments(with M.E variances obtained directly from data 3 via the technique in 2.4).

Parameters Estimated	Estimates Prior to M.E Adjustments	Error Adjusted estimates with M.E variance of X_1 = 0.64 and that of X_2 = 0.82	Error Adjusted estimates with M.E variance of X_1 = 0.64 V	Error Adjusted estimates with M.E variance of X_2 = 0.82
β_1	0.982(0.022)	0.427(0.032)	0.899(0.048)	1.011(0.023)
β_2	0.339 (0.011)	0.812(0.014)	0.433(0.015)	0.862(0.027)
Coefficient of variation (CV) for β_1	0.022	0.052	0.053	0.023
Coefficient of variation (CV) for β_2	0.032	0.017	0.034	0.031
Model Deviance	3366	-17688	-11827	-16767
Residual variance	0.113	0.001	0.004	0.001

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In Tables 3 – 5, it is discernable from the coefficient estimates of X_1 and X_2 that, adjusting for incidence of M.E's in both variables as they occur in the model does not, in general, necessarily yield coefficient estimates that are less biased than what obtains when adjustment for M.E's is made for only one of the variables. Indeed, adjusting for incidence of M.E's in both variables seems to either over estimate or underestimates the coefficients. For instance, the estimates of β_1 reduce by an average of 66% while the coefficient of variation increases by (an incredible) average of 135% in a situation where adjustments are made for incidence of M.E's in both X_1 and X_2 . On the other hand, we find the estimates of β_1 reducing, on average, by 14% and the coefficients of variation reducing, on average, by 30% if only X_1 is adjusted for incidence of M.E's. In a similar vein, the estimates of β_2 increasing, on average by 123% with the coefficients of variation increasing by 10% for a situation where both variables (X_1 and X_2) were adjusted for M.E's and these estimates increased, on average by 6% but the coefficients of variation reduced by 22% in the case where only X_1 was adjusted for incidence of M.E's.

If only X_2 is adjusted for incidence of M.E's, we find that, on average, estimates of β_1 decrease by 6% and coefficients of variation increase, on average, by 34% while estimates of β_2 increase by 141% and the coefficients of variation decrease by 14%. In general, however, model deviances and residual variances reduce when adjustments are made for errors. The point made here is that, in general, adjusting for the incidence of M.E's simultaneously in all perceived error-prone variables in a model will not necessarily yield better (less biased) coefficient estimates than what obtains when adjustments are made for the errors in only one of the variables.

Comparing the tabular values in Tables 3-5 with those of Table 6 reveals that employing the M.E variances obtained in data 3 via the iterative technique in 2.4 (i.e 0.64 for X_1 and 0.82 for X_2) to adjust for incidence of M.E's, does not give coefficient estimates that have , on average, less coefficient of variation values than what obtains when we employ the M.E variance values of 0.26 for X_1 and 0.46 for X_2. This may probably be attributed to weaknesses associated with the simulation technique that ultimately generated highly error-prone and unrealistic values of data 3. It is opined that if the values in a data set are highly error prone and unrealistic, then any M.E variance estimates of variables associated with them, using the iterative technique, will likely be suspect.

CONCLUSION

In fixed effects models such as linear and generalized linear models, there are several literature that elaborate the consequences of measurement errors on model adequacy especially in situations where such errors are apparent and non-ignorable. The efficacy of mixed effects models such as multilevel (hierarchical) linear models is also adversely affected by a failure to properly account for measurement errors in their formulation and/or estimation. At the crux of measurement error adjustment approaches in the affected models is the need to properly estimate the measurement error variances and reliabilities of the error-prone variables in the models. In general, the behaviour of biases associated with measurement error in covariates or the response for mixed effects models (such as multilevel linear models) could be complex and so attempts are currently being made to proffer ways of realistically adjusting for incidence of measurement errors in such models. The efficacy of an iterative measurement error adjustment technique was examined within the frame work of a 2-level model. In the illustrative analysis, two explanatory variables perceived error-prone were considered. It was found that (i) coefficient estimates generally disattenuated when adjustments were made for the incidence of errors in the explanatory variables (ii) adjusting for incidence of measurement errors in both variables simultaneously in the model did not necessarily yield less biased coefficient estimates (as proxied by their coefficients of variation) than what obtained when only one variable was adjusted for error (iii) adjustments for incidence of measurement errors in a situation where data used was likely generated via a defective simulation approach did not yield less biased coefficient estimates than what obtained using more realistic data (iv) adjusting for incidence of measurement errors however (as expected) yielded models with reduced deviances and residual variances.

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